

recurrence evaluate terms

1. (a) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2 . 1
- (b) A second sequence is given by $4, 5, 7, 11, \dots$.
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
Find the values of p and q . 3
- (c) Either the sequence in (a) or the sequence in (b) has a limit.
(i) Calculate this limit.
(ii) Why does the other sequence not have a limit? 3

| Part | Marks | Level | Calc. | Content | Answer | U1 OC4 |
|------|-------|-------|-------|----------|--|------------|
| (a) | 1 | C | CN | A11 | $u_1 = 8, u_2 = -4$ | 2011 P2 Q3 |
| (b) | 3 | C | CN | A11, A10 | $p = 2, q = -3$ | |
| (c) | 3 | C | CN | A12, A13 | (i) $l = 0$ (ii) outside $-1 < p < 1$ | |

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| <ul style="list-style-type: none"> •¹ pd: find terms of a sequence •² ic: interpret sequence •³ ss: solve for one variable •⁴ pd: state second variable •⁵ ss: know how to find valid limit •⁶ pd: calculate a valid limit only •⁷ ic: state reason | <ul style="list-style-type: none"> •¹ $u_1 = 8$ and $u_2 = -4$ •² e.g. $4p + q = 5$ and $5p + q = 7$ •³ $p = 2$ or $q = -3$ •⁴ $q = -3$ or $p = 2$ •⁵ $l = -\frac{1}{2}l$ or $l = \frac{0}{1 - (-\frac{1}{2})}$ •⁶ $l = 0$ •⁷ outside interval $-1 < p < 1$ |
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[SQA]

2. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

- (a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let u_n and u_{n+1} represent the amounts that he owes at the start of two successive months. Write down a recurrence relation involving u_{n+1} and u_n .

2

- (b) Find the date and the amount of the final payment.

4

| Part | Marks | Level | Calc. | Content | Answer | U1 OC4 |
|------|-------|-------|-------|----------|--|---------|
| (a) | 2 | C | CN | A10, A14 | $u_{n+1} = 1.015u_n - 300, u_0 = 2500$ | 1 P2 Q3 |
| (b) | 4 | C | CR | A11, A14 | 1 December, £290.68 | |

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| <ul style="list-style-type: none"> •¹ ic: interpret 1.5% •² ic: state the recurrence relation •³ ss: use recurrence relation •⁴ pd: process •⁵ ic: start final date •⁶ pd: process final payment | <ul style="list-style-type: none"> •¹ 1.015 <i>stated or implied by the start of (b)</i> •² $u_{n+1} = 1.015u_n - 300$ and initial value (e.g. $u_0 = 2500$) <i>stated or implied by the start of (b)</i> •³ u_1 i.e. £2237.50 •⁴ u_2 and u_3 i.e. £1971.06, £1700.63 •⁵ £286.38 •⁶ £290.68 for December payment |
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[END OF QUESTIONS]